GIRRAWEEN HIGH SCHOOL

MATHEMATICS (ADVANCED)

YEAR 12 - PRACTICE TRIAL HSC

Year 12 Mathematics (Advanced)
September 2021

TIME ALLOWED – 3 hours+ 10 min reading time TOTAL MARK – 100 marks

Instructions:

- Write using black pen.
- Calculators approved by NESA may be used.
- This task is open book but NOT open technology (ICT).
- For all questions, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.

You must have your:

- camera on (no virtual background)
- microphone off
- speakers on
- phone on the desk facing upwards and on silent and be visible to the teacher
- Upload your answers as a single PDF in your google classroom in the assignment folder.

Year 12 Mathematics Advanced outcomes

A student:

MA12-1	uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts
MA12-2	models and solves problems and makes informed decisions about financial situations using mathematical reasoning and techniques
MA12-3	applies calculus techniques to model and solve problems
MA12-4	applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems
MA12-5	applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs
MA12-6	applies appropriate differentiation methods to solve problems
MA12-7	applies the concepts and techniques of indefinite and definite integrals in the solution of problems
MA12-8	solves problems using appropriate statistical processes
MA12-9	chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use
MA12-10	constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

Allow about 15 minutes for this section
Select the right answer for questions 1-10

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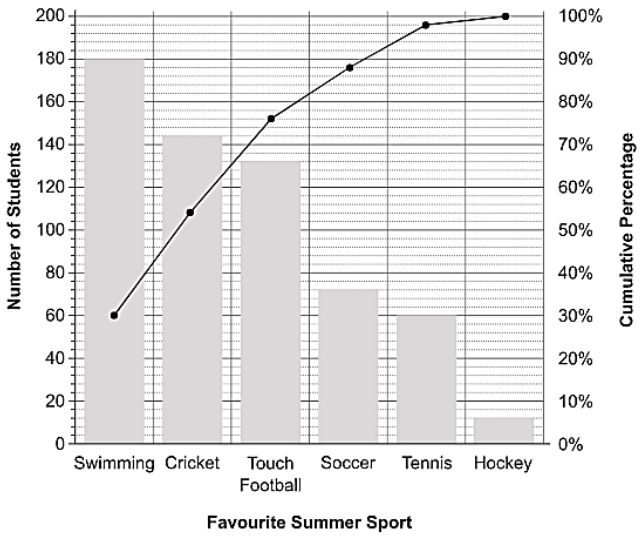
1. What is the value of $8e^{-2}$ correct to 2 significant figures?

A. 1.08
B. 1.083
C. 1.082
D. 1.1

---

2. A high school conducted a survey asking students what their favourite Summer sport was. The Pareto chart shows the data collected.



What percentage of the students chose Touch Football as their favourite summer sport?

A. 76%
B. 22%
C. 66%
D. 80%

---

3. How many terms are there in the series 486 + 162 + 54 + ……+ $\frac{2}{3}$

A. 5
B. 6
C. 7
D. 8

---

4. What is the derivative of $\ln(\sin x)$?

A. $\tan x$
B. $cosec\ x$
C. $\cot x$
D. $\ln(\cos x)$

5. Which of the following is NOT an even function?

A. $f(x) = (x-2)^2$

B. $f(x) = |x|$

C. $f(x) = \cos x$

D. $(x) = e^x + e^{-x}$

---

6. A discrete random variable X has the following probability distribution:

x	0	1	2	3
$P(X = x)$	$2q$	$6q$	$3q$	$4q$

The mean of X is which of the following?

A. 1

B. $\frac{8}{5}$

C. $\frac{1}{15}$

D. 2

---

7. The graph $y = x^2 + 4x + 7$ is reflected in the *y*-axis, followed by a translation of 2 units to the right. What is the equation of the new graph?

A. $y = (-x-2)^2 + 4(-x-2) + 7$

B. $y = (-x+2)^2 + 4(-x+2) + 7$

C. $-y = (x+2)^2 + 4(x+2) + 7$

D. $-y = (x-2)^2 + 4(x-2) + 7$

---

8. Given $f(x) = \log_e(x-3)$ and $g(x) = x + 4$, the domain of $f\big(g(x)\big)$ is

A. $x \geq -4$

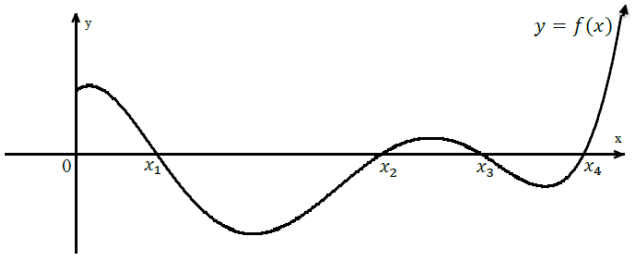
B. $x > -1$

C. $x > 3$

D. $x \geq 3$

---

9. The graph of the function $y = f(x)$ is shown below.



Given that $\int_0^{x_2} f(x)dx = -3$, $\int_0^{x_3} f(x)dx = -1$, and $\int_0^{x_4} f(x)dx = -4$,

What is the value of $\int_{x_2}^{x_4} f(x)dx$?

A. -1
B. -2
C. -3
D. -4

---

10. If $f(x) = 3\log_e(2x)$, $and\ f(5x) = \log_e(y)$, then y equal to

A. $30x$
B. $6x$
C. $125x^3$
D. $1000x^3$

Answer the questions in the booklet provided.
Your responses should include relevant mathematical reasoning and/ or calculations.

Question 11 (6 marks)

a) Find $\int \frac{2x+5}{x^2} dx$ 3m

b) Solve $|3x + 2| < 5$ and draw the solution on the number line. 3m

Question 12 (8 marks)

i. Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 3$. Leave your answer in terms of e. 2m

ii. Find the coordinates of the point where the tangent meets the x-axis. 2m

iii. Find the coordinates of the point where the tangent meets the y-axis. 2m

iv. Find the exact area of the triangle formed by this tangent and the axes. 2m

Question 13 (7 marks)

a) Solve $\sqrt{2}\sin x + 1 = 0$ for the domain $0 \leq x \leq 2\pi$ 2m

b)

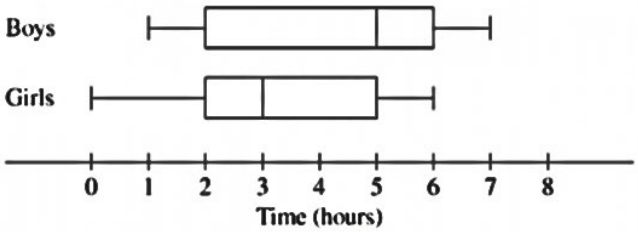
i. Give exact value of $\cos 150°$ 1m

ii. In ΔPQR, $PR = 5\sqrt{3}$ cm, $QR = 11$cm and $\angle PRQ = 150°$. Find the length of side PQ. 2m

iii. Find $\cos P$ 2m

Question 14 (3 marks)

In a school, boys and girls were surveyed about the time they usually spend on the internet over a weekend. These results were displayed in box and whisker plots, as shown below.



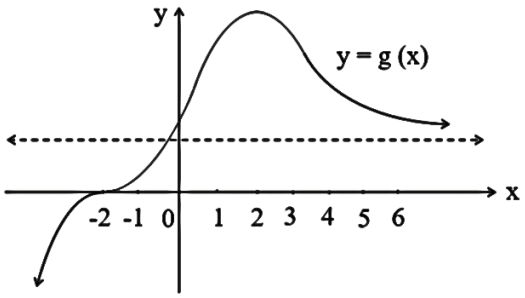
i. Find the interquartile range for boys. 1m

ii. What percentage of girls spend 5 or less hours on the internet over a weekend? 1m

iii. Jenny said that the graph shows that the same number of boys as girls usually spend between 5 and 6 hours on the internet over a weekend.
Under what circumstances would this statement be true? 1m

Question 15 (6 marks)

a) The diagram shows the graph of $y = g(x)$. Sketch the graph of $y = g'(x)$ 3m



b) Use the trapezoidal rule with four subintervals, together with appropriate log laws, to show that 3m

$$\int_1^5 \ln x \, dx \approx \ln 54$$

Question 16 (6 marks)

a) Find the sum to infinity of the series 2m
$32 + 16 + 8 + 4 + \cdots \ldots \ldots \ldots$

b) A quantity Q of radium at time t years is given by $Q = Q_0 e^{-kt}$, where k is a positive constant and Q_0 is the amount of radium at time $t = 0$.

i. Given that by $Q = \frac{1}{2}Q_0$ when $t = 1690$ years, calculate the value of k. 2m

ii. After how many years does only 20% of the initial amount of radium remains? Give your answer correct to the nearest year. 2m

Question 17 (6 marks)

a) Sketch the graph of $y = 10\sin\frac{\pi}{6}x$ and $y = x$ for $0 \leq x \leq 12$ in the space provided below. Mark the point of intersection at $(5,5)$. 3m

b) Find a simplified exact expression for the area bounded by the two graphs for $0 \leq x \leq 5$. 3m

Question 18 (8 marks)

a) The curve $y = ax^2 + bx + 3$ has a turning point at $(-1,0)$. Find the values of a and b. 3m

b) A coal chute is built in the shape of an upturned cone. The sum of the base radius r and the height h is 12 metres. ($V = \frac{1}{3}\pi r^2 h$)

i. Show that the volume of the coal chute is given by $V = 4\pi r^2 - \frac{1}{3}\pi r^3$. 2m

ii. Find the radius of the cone that yields the maximum volume. 3m

Question 19 (6 marks)

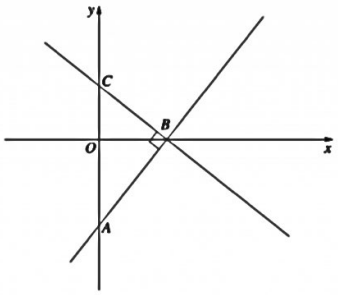
a) Find $\frac{d}{dx}\sqrt{5x^2 + 1}$. 2m

b) Hence find $\int \frac{x}{\sqrt{5x^2+1}}\,dx$ 1m

c) If $f'(x) = 6x^2 + 3$ and $f(2) = 8$, find $f(x)$ 3m

Question 20 (7 marks)

a) In the diagram, the points A and C lie on the y axis and the point B lies on the x axis. The line AB has equation $y = \sqrt{3}x - 3$. The line BC is perpendicular to AB.



i. Find the equation of the line BC. 2m

ii. Find the area of the triangle ABC 2m

b) Solve $e^{2\ln x} = x + 6$ 3m

Question 21 (4 marks)

Two boxes each contain four stones that differ only in colour.
Box 1 contains four black stones.
Box 2 contains two black stones and two white stones.
A box is chosen randomly, and one stone is drawn randomly from it.

Each box is equally likely to be chosen, as is each stone.

a) What is the probability that the randomly drawn stone is black? 2m

b) It is not known from which box the stone has been drawn. Given that the stone that is drawn is black, what is the probability that it was drawn from Box 1?

2m

Question 22 (5 marks)

i. Find the vertical and horizontal asymptote of the hyperbola $y = \frac{x-2}{x-4}$ and hence sketch the graph of $y = \frac{x-2}{x-4}$ showing all intercepts with the coordinate axes.

3m

ii. Hence or otherwise, find the values of x for which
$\frac{x-2}{x-4} \leq 3$ 2m

Question 23 (5 marks)

Ten high school students have their height and the length of their right foot measured. The results are recorded in the table below.

Height (cm)	142	164	183	172	154	158	149	181	175	177
Foot (cm)	20	26	28	26	22	23	24	29	27	27

i. Using calculator, calculate Pearson's correlation coefficient for the data. Give your answer correct to 3 decimal places. 2m

ii. Describe the strength of the association between height and length of right foot for these students. 1m

iii. Using calculator, determine the equation of least squares regression line that allows height to be predicted from right foot length. 2m

Question 24 (7 marks)

A particle moves on the x axis with velocity, v m/s, given at any time, t seconds, $t \geq 0$, by

$$v = \frac{1}{\sqrt{2t+1}}$$

Initially the particle is at the origin.

i. Find the initial velocity and the velocity after 12 seconds. 2m

ii. Sketch the velocity-time graph. 1m

iii. Find the acceleration of the particle after 12 seconds. 2m

iv. Find the displacement of the particle as a function of time. 2m

Question 25 (6 marks)

Between 5 am and 5 pm on 3rd March 2009, the height of the tide in a harbour was given by

$h = 1 + 0.7\sin\left(\frac{\pi}{6}t\right)$ for $0 \leq t \leq 12$

where h is in metres and t is in hours, with $t = 0$ at 5 am.

i. What is the period of the function h? 1m

ii. What was the value of h at low tide, and at what time did low tide occur? 2m

iii. A ship is able to enter the harbour only if the height of the tide is at least 1.35m.
Find all times between 5 am and 5 pm on 3rd March 2009 during which the ship was able to enter the harbour. 3m

END OF TEST

2021 – Practice Trial Solutions

GIRRAWEEN HIGH SCHOOL
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Marks:
/10

Booklet 1 (Q1 to Q10)

Student Number:

Allow about 15 minutes for this section
Use the multiple-choice answer sheet for questions 1-10

1. What is the value of $8e^{-2}$ correct to 2 significant figures?

A. 1.08
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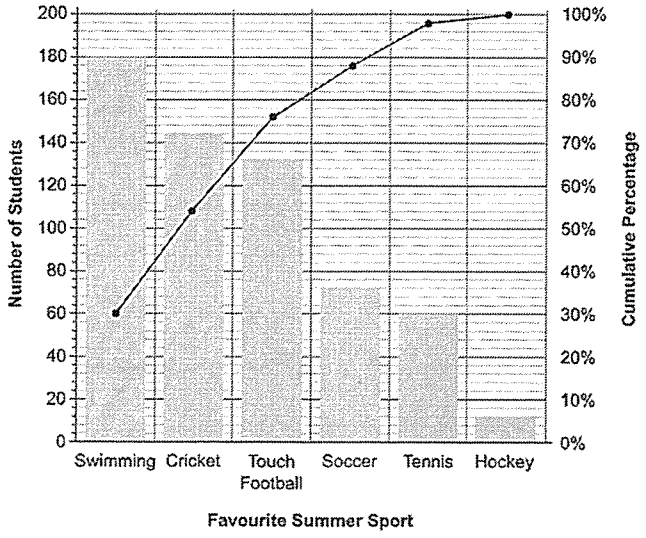
2. A high school conducted a survey asking students what their favourite Summer sport was. The Pareto chart shows the data collected.



What percentage of the students chose Touch Football as their favourite summer sport?

A. 76%
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C. 66%
D. 80%

3. How many terms are there in the series 486 + 162 + 54 + ...+ $\frac{2}{3}$

A. 5
B. 6
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4. What is the derivative of $\ln(\sin x)$?

A. $\tan x$
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5. Which of the following is NOT an even function?

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B. $f(x) = |x|$
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D. $(x) = e^x + e^{-x}$

---

6. A discrete random variable X has the following probability distribution:

x	0	1	2	3
$P(X = x)$	$2q$	$6q$	$3q$	$4q$

The mean of X is which of the following?

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B. $\frac{8}{5}$
C. $\frac{1}{15}$
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7. The graph $y = x^2 + 4x + 7$ is reflected in the y-axis, followed by a translation of 2 units to the right. What is the equation of the new graph?

A. $y = (-x-2)^2 + 4(-x-2) + 7$
B. $y = (-x+2)^2 + 4(-x+2) + 7$
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8. Given $f(x) = \log_e(x-3)$ and $g(x) = x+4$, the domain of $f(g(x))$ is

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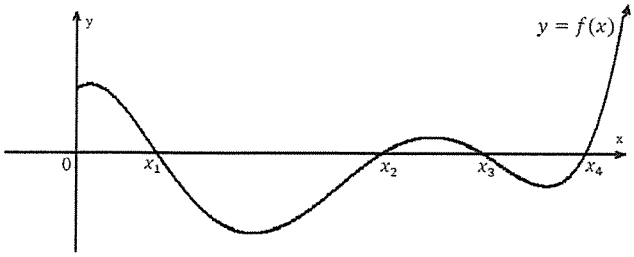

Given that $\int_0^{x_2} f(x)dx = -3$, $\int_0^{x_3} f(x)dx = -1$, and $\int_0^{x_4} f(x)dx = -4$,

What is the value of $\int_{x_2}^{x_4} f(x)dx$?

A. -1
B. -2
C. -3
D. -4

---

10. If $f(x) = 3\log_e(2x)$, *and* $f(5x) = \log_e(y)$, then y equal to

A. $30x$
B. $6x$
C. $125x^3$
D. $1000x^3$

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Booklet 2 (Q11,12,13)

Marks:	
Q11	/6
Q12	/8
Q13	/7
Total	/21

Student Number:

Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/ or calculations.

Question 11 (6 marks)

a) Find $\int \frac{2x+5}{x^2} dx$ 3m

$= \int \frac{2}{x} + 5x^{-2} \, dx$

$= 2\ln|x| + \frac{5x^{-1}}{-1} + C$

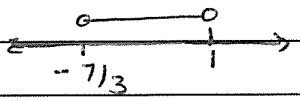
$= 2\ln|x| - \frac{5}{x} + C$

b) Solve $|3x + 2| < 5$ and draw the solution on the number line. 3m

$3x+2 < 5$ $\quad$ $3x+2 > -5$

$3x < 3$ $\quad$ $3x > -7$

$x < 1$ $\quad$ $x > -7/3$



Question 12 (8 marks)

i. Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 3$. Leave your answer in terms of e. 2m

$\frac{dy}{dx} = 2e^{2x}$ when $x = 3$

$y = e^6$

$m_t = \left.\frac{dy}{dx}\right|_{x=3} = 2e^6$

Equation of the tangent

$y - e^6 = 2e^6(x-3)$

$y = 2e^6x - 5e^6$ — (A)

ii. Find the coordinates of the point where the tangent meets the x-axis. 2m

Sub $y = 0$ into (A)

$2e^6x - 5e^6 = 0$

$x = 5/2$

$(5/2, 0)$

iii. Find the coordinates of the point where the tangent meets the y-axis. 2m

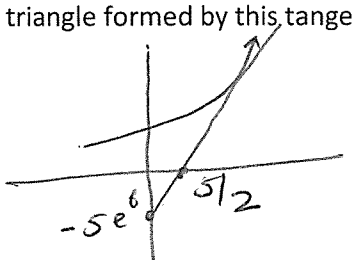
Sub $x = 0$ into (A)

$y = -5e^6$

$(0, -5e^6)$

exact

iv. Find the area of the triangle formed by this tangent and the axes. 2m



$$\text{Area} = \frac{1}{2} \times 5/2 \times 5e^6$$

$$= \frac{25e^6}{4} \ u^2$$

Question 13 (7 marks)

a) Solve $\sqrt{2}\sin x + 1 = 0$ for the domain $0 \leq x \leq 2\pi$ 2m

$$\sin x = -\frac{1}{\sqrt{2}}$$

S	A
T	C ✓

$$x = \pi + \frac{\pi}{4}, \ 2\pi - \frac{\pi}{4} \quad ✓$$

$$x = \frac{5\pi}{4}, \ \frac{7\pi}{4}$$

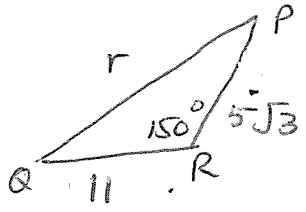
b)

i. Give exact value of $\cos 150°$ 1m

$$\cos(180 - 30) = -\cos 30$$

$$= -\frac{\sqrt{3}}{2}$$

ii. In ΔPQR, $PR = 5\sqrt{3}$ cm, $QR = 11$cm and $\angle PRQ = 150°$. Find the length of side PQ. 2m



$r = \sqrt{(5\sqrt{3})^2 + 11^2 - 2 \times 5\sqrt{3} \times 11 \times \cos 150}$

$r = 19$ cm

iii. Find $\cos P$ 2m

$\cos P = \dfrac{19^2 + (5\sqrt{3})^2 - 11^2}{2 \times 19 \times 5\sqrt{3}}$

$\cos P = \dfrac{315}{190\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{315\sqrt{3}}{190 \times 3}$

$\cos P = \dfrac{21\sqrt{3}}{38}$

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Marks:	
Q14	/3
Q15	/6
Q16	/6
Total	/15

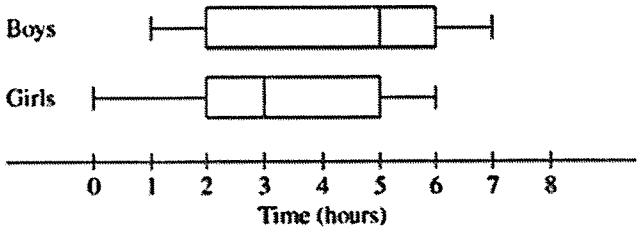
Booklet 3 (Q14,15,16)

Student Number:

Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/ or calculations.

Question 14 (3 marks)

In a school, boys and girls were surveyed about the time they usually spend on the internet over a weekend. These results were displayed in box and whisker plots, as shown below.



i. Find the interquartile range for boys. 1m

$6 - 2 = 4$

ii. What percentage of girls spend 5 or less hours on the internet over a weekend? 1m

75 %

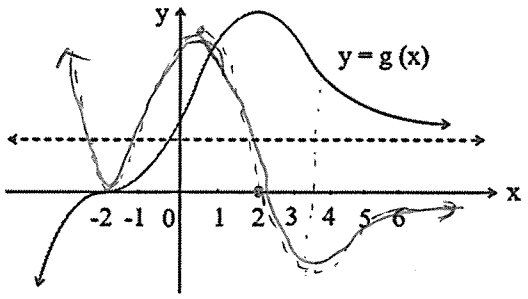
iii. Jenny said that the graph shows that the same number of boys as girls usually spend between 5 and 6 hours on the internet over a weekend.
Under what circumstances would this statement be true? 1m

5-6 hours for girls accounts for 25% of all girls
5-6 hours for boys accounts for 25% of all boys

This will only be same number if the number of all girls surveyed equal the number of boys surveyed.

Question 15 (6 marks)

a) The diagram shows the graph of $y = g(x)$. Sketch the graph of $y = g'(x)$ 3m



b) Use the trapezoidal rule with four subintervals, together with appropriate log laws, to show that 3m

$$\int_1^5 \ln x \, dx \approx \ln 54$$

x	1	2	3	4	5
$\ln x$	0	$\ln 2$	$\ln 3$	$\ln 4$	$\ln 5$

$$\int_1^5 \ln x \, dx = \frac{1}{2}\left[(0 + \ln 5) + 2(\ln 2 + \ln 3 + \ln 4)\right]$$

$$= \frac{1}{2}\left[\ln 5 + 2\ln(2 \times 3 \times 4)\right]$$

$$= \frac{1}{2}\left[\ln 5 + \ln 24^2\right]$$

$$= \frac{1}{2}\left[\ln 5 + \ln 576\right]$$

$$= \frac{1}{2}\left[\ln(5 \times 576)\right] = \frac{1}{2}\ln 2880$$

$$= \ln \sqrt{2880}$$

$$\doteq \ln 54$$

Question 16 (6 marks)

a) Find the sum to infinity of the series 2m

$32 + 16 + 8 + 4 + \cdots \ldots \ldots \ldots$

$a = 32, \quad r = \frac{1}{2}$

$$S_\infty = \frac{32}{1 - 1/2} = 64$$

b) A quantity Q of radium at time t years is given by $Q = Q_0 e^{-kt}$, where k is a positive constant and Q_0 is the amount of radium at time $t = 0$.

i. Given that by $Q = \frac{1}{2}Q_0$ when $t = 1690$ years, calculate the value of k. 2m

$0.5 = e^{-k \times 1690}$

$-1690k = \log_e 0.5$

$k = \frac{\ln 0.5}{-1690}$

$k = 0.0004101461626$

ii. After how many years does only 20% of the initial amount of radium remains? Give your answer correct to the nearest year. 2m

$0.2 = e^{-kt}$

$-kt = \ln 0.2$

$t = \frac{\ln 0.2}{-k}$

$t = \ln 0.2 \div \left(\frac{\ln 0.5}{1690}\right)$

$= 3831$ years.

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Booklet 4 (Q17,18,19)

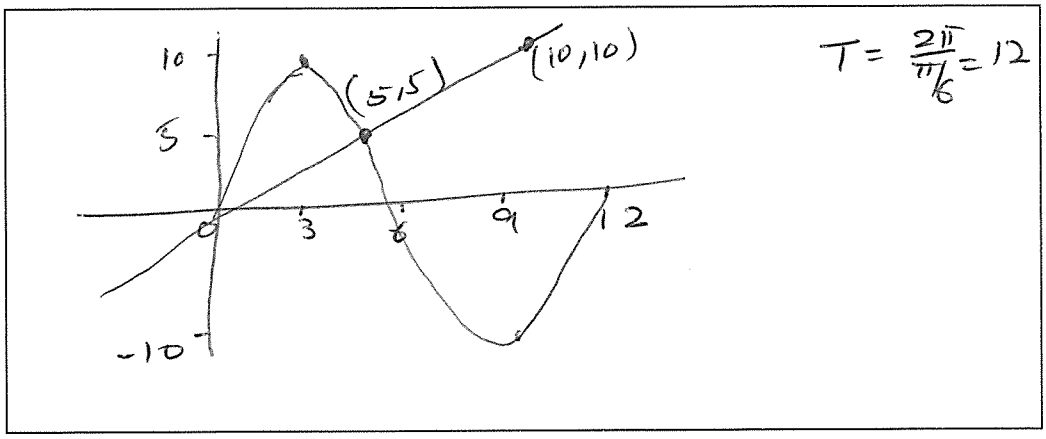
Marks:	
Q17	/6
Q18	/8
Q19	/5
Total	/19

Student Number:

Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/ or calculations.

Question 17 (6 marks)

a) Sketch the graph of $y = 10\sin\frac{\pi}{6}x$ and $y = x$ for $0 \leq x \leq 12$ in the space provided below. Mark the point of intersection at (5,5). 3m



b) Find a simplified exact expression for the area bounded by the two graphs for $0 \leq x \leq 5$. 3m

$$A = \int_0^5 \left(10\sin\frac{\pi}{6}x - x\right)dx$$

$$= \left[10x - \frac{6}{\pi}\cos\frac{\pi x}{6} - \frac{x^2}{2}\right]_0^5$$

$$= \left[-\frac{60}{\pi}\cos\frac{\pi x}{6} - \frac{x^2}{2}\right]_0^5$$

$$= \left[-\frac{60}{\pi}\cos\frac{5\pi}{6} - \frac{25}{2}\right] - \left[-\frac{60}{\pi}\cos 0 - 0\right]$$

$$= -\frac{60}{\pi} \times \left(\frac{-\sqrt{3}}{2}\right) - \frac{25}{2} + \frac{60}{\pi}$$

$$= \frac{30\sqrt{3}}{\pi} + \frac{60}{\pi} - \frac{25}{2}$$

$$= \frac{60\sqrt{3} + 120 - 25\pi}{2\pi}$$

Question 18 (8 marks)

a) The curve $y = ax^2 + bx + 3$ has a turning point at $(-1,0)$. Find the values of a and b. 3m

$y' = 2ax + b = 0$ at the stationary point

Sub $x = -1$

$-2a + b = 0$ — ① $\quad b = 2a$

$(-1,0)$ lies on $y = ax^2 + bx + 3$

$0 = a - b + 3$

$a - b = -3$

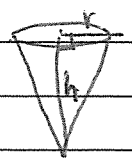
from (i) $\quad a - 2a = -3$

$a = 3$

$b = 6$

b) A coal chute is built in the shape of an upturned cone. The sum of the base radius r and the height h is 12 metres. ($V = \frac{1}{3}\pi r^2 h$)

i. Show that the volume of the coal chute is given by $V = 4\pi r^2 - \frac{1}{3}\pi r^3$. 2m



$r + h = 12$

$h = 12 - r$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (12 - r)$

$= 4\pi r^2 - \frac{1}{3}\pi r^3$

ii. Find the radius of the cone that yields the maximum volume. 3m

$\frac{dV}{dr} = 8\pi r - \pi r^2$

Solve $\frac{dV}{dr} = 0$ for max, min

$8\pi r - \pi r^2 = 0$

$\pi r(8 - r) = 0$

$r = 0, \quad r = 8$

$r > 0 \quad \therefore r = 8$ m

$\frac{d^2V}{dr^2} = 8\pi - 2\pi r$

$\left.\frac{d^2V}{dr^2}\right|_{r=8} = 8\pi - 2\pi \times 8$

$= -8\pi < 0$

$\therefore \quad r = 8$ m gives maximum volume.

Question 19 (5 marks)

a) Find $\frac{d}{dx}\sqrt{5x^2 + 1}$. 1m

$= \frac{d}{dx}(5x^2+1)^{1/2}$

$= \frac{1}{2}(5x^2+1)^{-\frac{1}{2}} \times 10x$

$= \frac{5x}{\sqrt{5x^2+1}}$

b) Hence find $\int \frac{x}{\sqrt{5x^2+1}}\,dx$ 1m

$$\int \frac{5x}{\sqrt{5x^2+1}}\,dx = \sqrt{5x^2+1} + C_1 \text{ from (a)}$$

Divide by 5

$$\therefore \int \frac{x}{\sqrt{5x^2+1}}\,dx = \frac{1}{5}\sqrt{5x^2+1} + C_2$$

c) If $f'(x) = 6x^2 + 3$ and $f(2) = 8$, find $f(x)$ 3m

$$f(x) = \int (6x^2+3)\,dx$$

$$= 2x^3 + 3x + C$$

$$f(2) = 16 + 6 + C = 8$$

$$C = -14$$

$$\therefore f(x) = 2x^3 + 3x - 14$$

Marks:	
Q17	/7
Q18	/4
Q19	/5
Total	/16

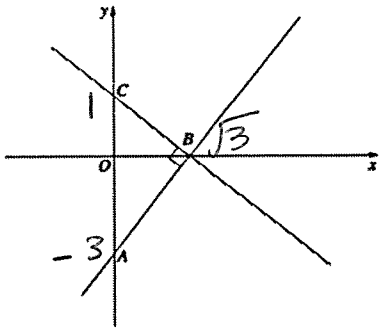
Booklet 5 (Q20,21,22)

Student Number:

Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/ or calculations.

Question 20 (7 marks)

a) In the diagram, the points A and C lie on the y axis and the point B lies on the x axis. The line AB has equation $y = \sqrt{3}x - 3$. The line BC is perpendicular to AB.



i. Find the equation of the line BC. 2m

x-int. of AB

$\sqrt{3}x - 3 = 0$

$x = \frac{3}{\sqrt{3}}$

$x = \sqrt{3}$

gradient of AB $= \sqrt{3}$

gradient of BC $= -\frac{1}{\sqrt{3}}$

Equation of BC

$y - 0 = -\frac{1}{\sqrt{3}}(x - \sqrt{3})$

$y = -\frac{1}{\sqrt{3}}x + 1$

ii. Find the area of the triangle ABC 2m

y-int. of AB = 1

Area of $\triangle ABC = \frac{1}{2} \times 4 \times \sqrt{3}$

$= 2\sqrt{3}\ u^2$

b) Solve $e^{2\ln x} = x + 6$ 3m

$2\ln x = \ln_e (x+6)$

$\ln x^2 = \ln (x+6)$

$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ or $x = -2$

$x > 0$

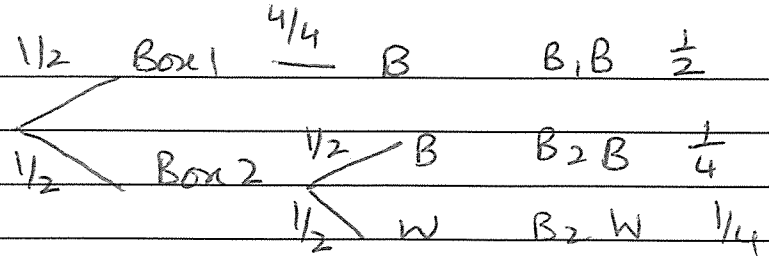
$\therefore x = 3$

Question 21 (4 marks)

Two boxes each contain four stones that differ only in colour.
Box 1 contains four black stones.
Box 2 contains two black stones and two white stones.
A box is chosen randomly, and one stone is drawn randomly from it.

Each box is equally likely to be chosen, as is each stone.

a) What is the probability that the randomly drawn stone is black? 2m



$P(\text{Black}) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

OR $P(B) = 1 - P(W) = 1 - \frac{1}{4} = \frac{3}{4}$

b) It is not known from which box the stone has been drawn. Given that the stone that is drawn is black, what is the probability that it was drawn from Box 1?

2m

$P(\text{Box} \mid \text{Black}) = \frac{1}{2} \div \frac{3}{4}$

$= \frac{2}{3}$

Question 22 (5 marks)

i. Find the vertical and horizontal asymptote of the hyperbola $y = \frac{x-2}{x-4}$ and hence sketch the graph of $y = \frac{x-2}{x-4}$ showing all intercepts with the coordinate axes.

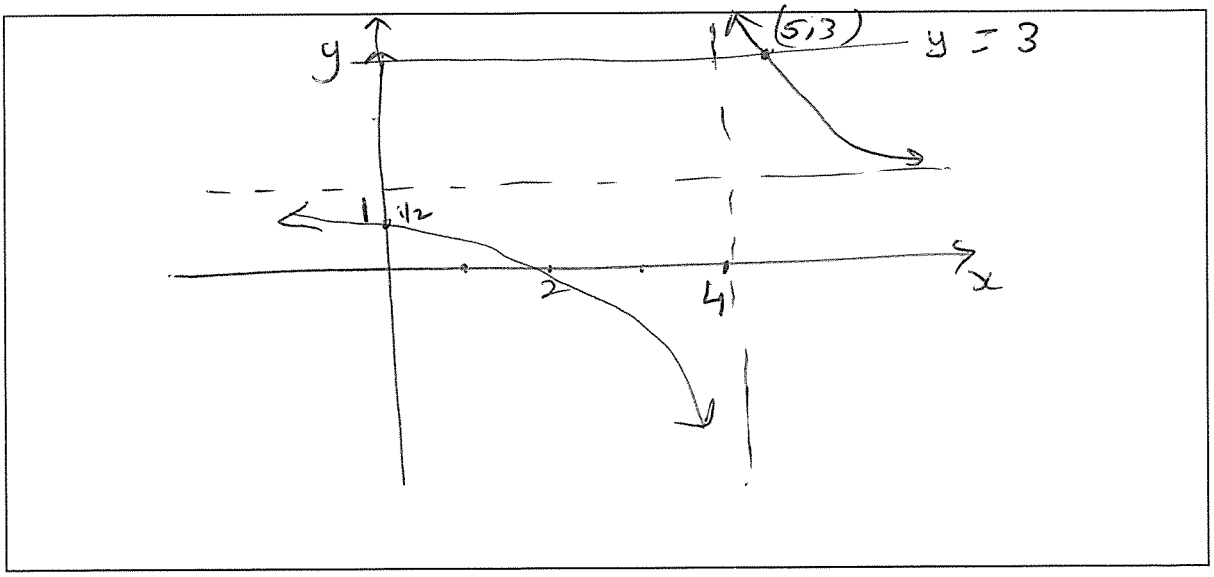
3m

Vertical asymptote at $x = 4$

$\lim_{x \to \infty} \frac{x-2}{x-4} = \lim_{x \to \infty} \frac{1 - \frac{2}{x}}{1 - \frac{4}{x}} = 1$

y-int $= \frac{1}{2}$

x int $= 2$



ii. Hence or otherwise, find the values of x for which

$\frac{x-2}{x-4} \leq 3$ 2m

Find x when $\frac{x-2}{x-4} = 3$

$3x - 12 = x - 2$

$2x = 10$

$x = 5$

$(5, 3)$ is the point of intersection.

$\therefore \frac{x-2}{x-4} \leq 3$ when $x < 4$ or $x \geq 5$

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Marks:	
Q23	/5
Q24	/7
Q25	/7
Total	/19

Booklet 6 (Q23,24,25)

Student Number:

Answer the questions in the spaces provided.
Your responses should include relevant mathematical reasoning and/ or calculations.

Question 23 (5 marks)

Ten high school students have their height and the length of their right foot measured. The results are recorded in the table below.

Height (cm)	142	164	183	172	154	158	149	181	175	177
Foot (cm)	20	26	28	26	22	23	24	29	27	27

i. Using calculator, calculate Pearson's correlation coefficient for the data. Give your answer correct to 3 decimal places. 2m

$r = 0.94095$

$= 0.941$ (3dpl.)

ii. Describe the strength of the association between height and length of right foot for these students. 1m

Strong positive

iii. Using calculator, determine the equation of least squares regression line that allows height to be predicted from right foot length. 2m

Foot length = x

Height = y

$y = 47.4 + 4.7x$

Question 24 (7 marks)

A particle moves on the x axis with velocity, v m/s, given at any time, t seconds, $t \geq 0$, by

$$v = \frac{1}{\sqrt{2t+1}}$$

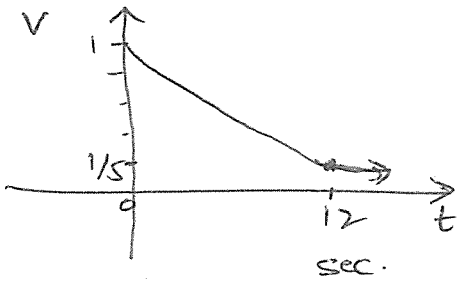
Initially the particle is at the origin.

i. Find the initial velocity and the velocity after 12 seconds. 2m

$t = 0$ $\quad$ $t = 12$

$v = 1 \text{ m/s}$ $\quad$ $v = \frac{1}{5} \text{ m/s}$

ii. Sketch the velocity-time graph. 1m



iii. Find the acceleration of the particle after 12 seconds. 2m

$$v = (2t+1)^{-1/2}$$

$$a = \frac{dv}{dt} = -\frac{1}{2}(2t+1)^{-3/2} \times 2$$

$$= -\frac{1}{(2t+1)^{3/2}}$$

at $t = 12$ $\quad a = -\frac{1}{125}\ m/s^2$

iv. Find the displacement of the particle as a function of time. 2m

$$x = \int (2t+1)^{-1/2}\, dt$$

$$= \frac{(2t+1)^{1/2}}{2 \times 1/2} + C$$

$$x = (2t+1)^{1/2} + C$$

When $t = 0$, $x = 0$

$$0 = 1 + C$$

$$C = -1$$

$\therefore x = \sqrt{2t+1} - 1$

Question 25 on the next page

Question 25 (6 marks)

Between 5 am and 5 pm on 3rd March 2009, the height of the tide in a harbour was given by

$h = 1 + 0.7\sin\left(\frac{\pi}{6}t\right)$ for $0 \le t \le 12$

where h is in metres and t is in hours, with $t = 0$ at 5 am.

i. What is the period of the function h? 1m

$T = \frac{2\pi}{\pi/6} = 12$

ii. What was the value of h at low tide, and at what time did low tide occur? 2m

h will be minimum when

$\sin\left(\frac{\pi}{6}.t\right) = -1$

$\therefore h = 1 + 0.7 \times -1$

$h = 0.3$ m

since $\sin x = -1$

$x = 3\pi/2$

$\frac{\pi}{6}t = 3\pi/2$

$t = 9$ hours

iii. A ship is able to enter the harbour only if the height of the tide is at least 1.35m.
Find all times between 5 am and 5 pm on 3rd March 2009 during which the ship was able to enter the harbour. 3m

Find t when $h \ge 1.35$ m

$1 + 0.7\sin\left(\frac{\pi}{6}t\right) \ge 1.35$

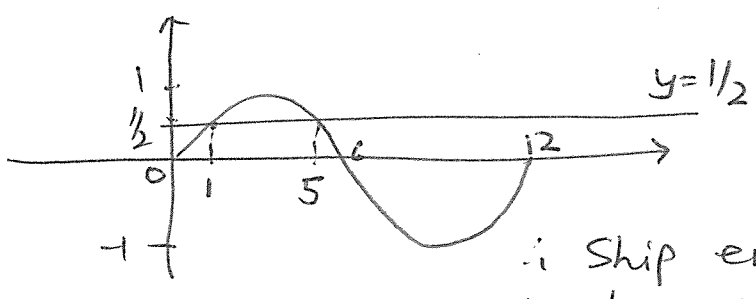
$0.7\sin\left(\frac{\pi}{6}t\right) \ge 0.35$

$\sin\frac{\pi}{6}t \ge \frac{1}{2}$

$\sin\frac{\pi}{6}t = \frac{1}{2}$ when

$\frac{\pi}{6}t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \ldots$

$t = 1, 5, 13, \ldots$



From the graph $\sin\left(\frac{\pi}{6}t\right) \ge \frac{1}{2}$ when $1 \le t \le 5$

∴ Ship enters the harbour between 6am and 10am.